Equilibrium with UDL (including partial UDL)

Calculate the reactions for a floor joist with uniformly distributed loading

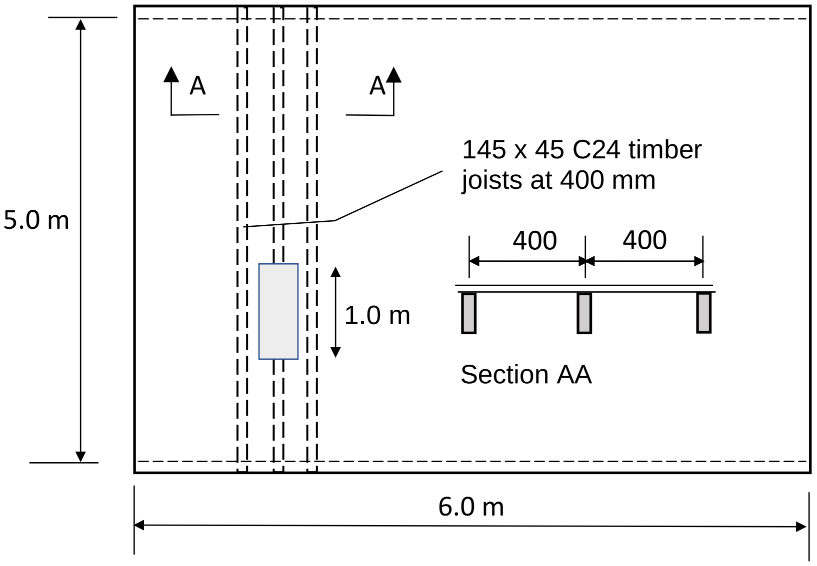


Figure 1 Floor plan

Defining the design load for a joist - Why a uniformly distributed load (UDL) is used

The plan above is for the floor of a room in a house. The flooring is supported by timber joists at 400 mm centres (i.e. the centre to centre distance between the joists is 400 mm). The joists are supported on walls. The joist spacing is governed by the span of the flooring that is normally 19mm planks or board. The live load on the floor (i.e. the load due to furniture, people, etc) will be highly variable so the simple assumption is made that the floor should be able to support a live load that is uniformly distributed over the whole area. Based on experience, codes of practice define what the values of these loads should be. For a floor in a house, a loading of 1.5 kN/sq m is normally used.

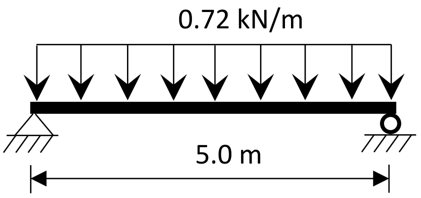
Assume that this is the design live load. Add 0.3 kN/sqm for dead load (i.e self-weight)
Total design load = 1.5 + 0.3 = 1.8 kN/sqm

The plan shows an area 1 metre length by 400 mm wide in the line of a joist. The load on this area will be 1.8*1.0*0.4 = 0.72 kN.
Therefore, if the floor loading is 1.8 kN/sqm the load on a joist will be a uniformly distributed load (UDL) per unit length of 0.72 kN/m.

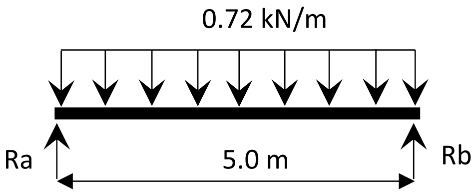
See more information about tributary areas here.

Calculate the reactions for the joist with UDL over full length

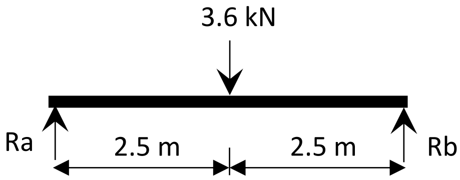
Calculate the reactions for a floor joist



Analysis model
UDL over full length of the joist.
Pin support on left and roller support on right.



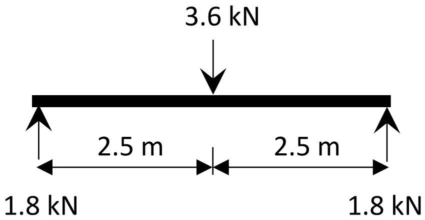
Free body diagram of the joist



Replace the UDL by its resultant
To calculate a moment due to the UDL, replace it by its resultant. The value of the resultant is the total load. This is calculated by multiplying the load per unit length by the length over which it acts- in this case:

Total load = 0.72*5.0 = 3.6 kN

The resultant acts at the centre of gravity of the load
By symmetry, the centre of gravity must be at the centre of the length of the loading, i.e. in the middle of the joist in this case.



Calculate the value of the reactions
Due to symmetry the reactions must be equal but to demonstrate the process, calculate them using the principle of equilibrium.
Apply moment equilibrium at the left support:

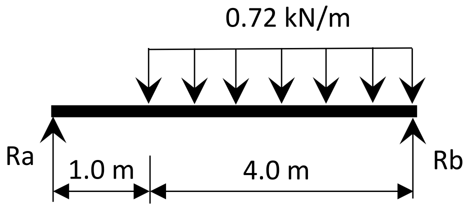
Σ M = 0
- 3.6*2.5 + Rb*5 = 0
Rb = (3.6*2.5)/5 = 1.8 kN

Apply vertical equilibrium:

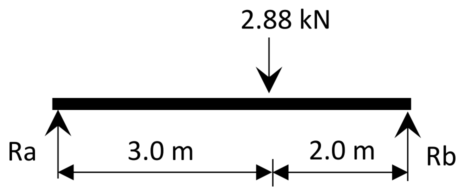
Σ Fy = 0
Ra + Rb - 3.6 = 0
Ra = 3.6 - Rb = 3.6 - 1.8 = 1.8 kN

Calculate the reactions for the joist with a partial UDL

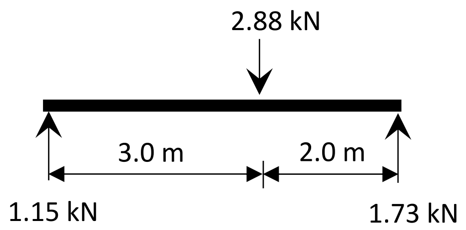
Assume that that one end of the joists extends one metre into an attic space where there will be no live load



Free body diagram
UDL over 4 m length of beam



Replace the UDL by its resultant
Total load = 0.72*4 = 2.88kN
Centre of gravity is at the centre of the length of the load i.e. 2.0 m from right hand end



Calculate the value of the reactions
Apply moment equilibrium at the left support:
$\Sigma M = 0$
$- 2.88*3.0 + Rb*5.0 = 0$
$Rb = 2.88*3.0/5 = 1.73$ kN
Apply vertical equilibrium:
$\Sigma Fy = 0$
$Ra + Rb – 2.88 = 0$
$Ra = 2.88 – Rb = 2.88 - 1.73 = 1.15$ kN

Process for calculating reactions due to an applied UDL:

1. Replace the UDL by its resultant whose value is the load per unit length times the length of the loaded part. The position of the resultant is in the middle of the loaded length.
2. Use the principle of equilibrium to calculate the reactions.

Note that when calculating internal force actions (shear force and bending moments) the replacement of the UDL by a single point load should not be made before a free body diagram of part of the structure is defined e.g. the bending moment for a beam with a central point load is twice that for a UDL of the same total value.

Metadata
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