Application Sheet

Roof Frame

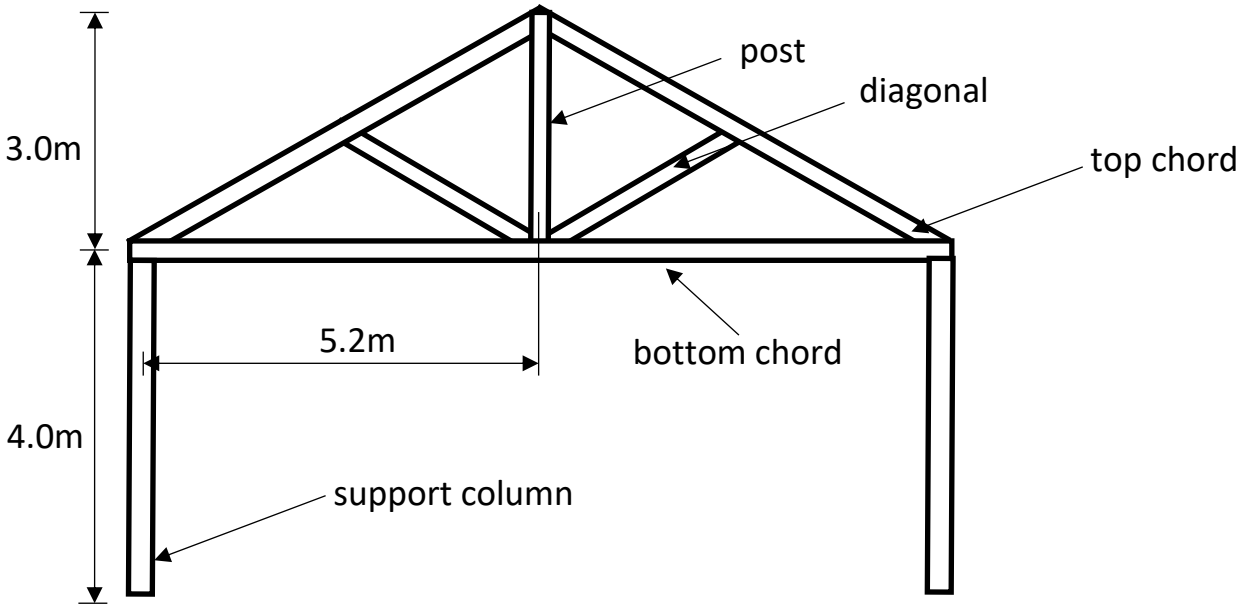
Context: Roof truss for a garden shed
Objective: Assessment of the frame for adequacy in compression

Concepts used in this application sheet

- Force: applied load, point load, internal force, resolution into components
- Equilibrium: equilibrium equation, force equilibrium
- Free Body Diagram

Timber frame (Photo by Arbre Évolution licensed under CC BY-SA 2.0)

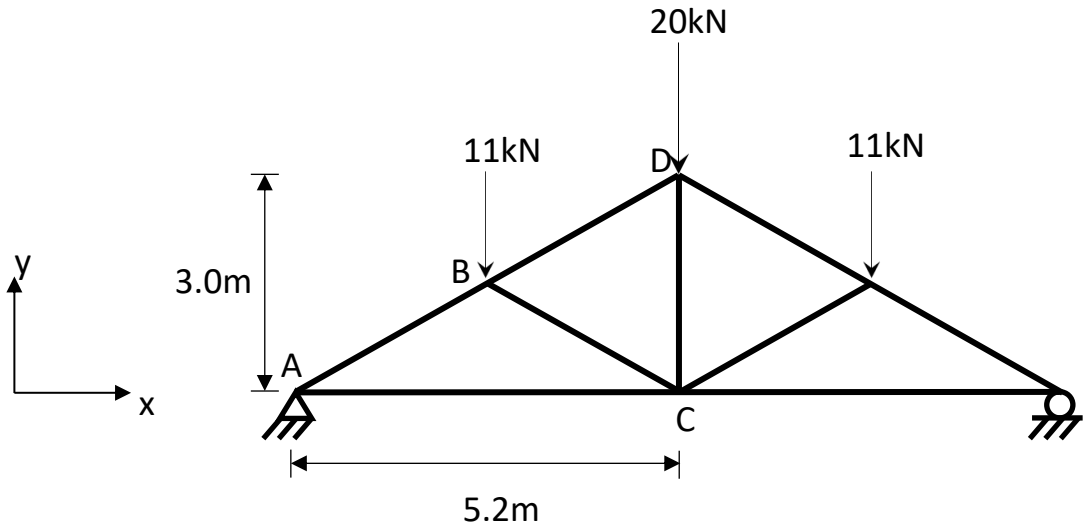
Engineering Model



All sections: 50x50mm timber

Structural Analysis

Analysis Model

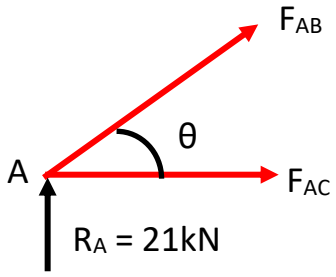


The frame is symmetric about DC

Calculations

Calculate the force in AB and AC

Calculate the support reaction at A, R_A:
Total load on the truss: 11+20+11 = 42kN
R_A = 42/2 = 21 kN (because the truss loading is symmetric)



Draw the free body diagram for point A:

θ = $\tan^{-1}$(DC/AC) = $\tan^{-1}$(3.0/5.2) = 30°
$\sum F_y$ = 21 + F_{AB}*sin(θ) = 0
F_{AB} = -21/sin(θ) = -21/sin(30°) = -42kN
F_{AB} is negative, i.e. it is compressive

$\sum F_x$ = F_{AB}*cos(θ) + F_{AC} = 0
F_{AC} = - F_{AB}*cos(θ) = -(-42)*cos(30) = 36.4kN (tensile)

Similarly, forces in all other members can be calculated:

Member	Force (kN)	Type
AB	-42	Compressive (strut)
AC	36.4	Tensile (tie)
BC	-11	Compressive (strut)
BD	-31	Compressive (strut)
CD	11	Tensile (tie)

Assessment

Using the allowable stress method
Strength criterion $\sigma/\sigma_a \leq 1.0$

- σ is the tensile or compressive stress due to loading (σ_t *or* σ_c)
- σ_a is the design allowable stress

Tensile assessment

Data input
Design allowable tensile stress of timber σ_a = 30N/mm^2 (from data booklet)
Maximum tensile force in the frame = 36.4kN
Section size = 50x50mm

Calculations
Working tensile stress in the frame = F/A = 36.4x10^3/(50x50) = 14.56N/mm^2

Apply the criterion
σ_t/σ_a = 14.56/30 = 0.49 < 1.0

Decision
The unity factor is less than one, hence this member is safe. As this is the largest tensile force on the frame, all other members will subsequently have an even higher FOS and thus all members in tension are safe.

Compressive assessment

Data input
Design allowable compressive stress of timber σ_a = 5N/mm^2
Maximum compressive force in the frame = 42kN
Section size = 50x50mm

Calculations
Working compressive stress in the frame = F/A = 42x10^3/(50x50) = 16.8N/mm^2

Apply the criterion
σ_c/σ_a = *16.8/5 = 3.36 >1.0*

Decision
The unity factor is greater than one and thus the beam is not safe

Final decision
As the FOS for the compressive member is less than one, the overall structure is not safe, and the member must be resized in order to ensure the safety of the structure.

Validation

Bending in members: The rafter members are continuous at joint B and the lower chord members are continuous at joint C. There will therefore be in bending in these members. The bending stresses will be low as compared with the axial stress and it is conventional to neglect such bending for frames of this type.

Roller support: It is likely that there will be some restraint to horizontal movement between the support points but it is conservative to neglect it.

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