Nodal analysis

Nodal analysis is used to determine the axial forces in the members of a pin-jointed frame structure. It involves applying the principle of equilibrium to successive joints in the structure.

Figure 1 Roof truss
(ARBRE EVOLUTION,
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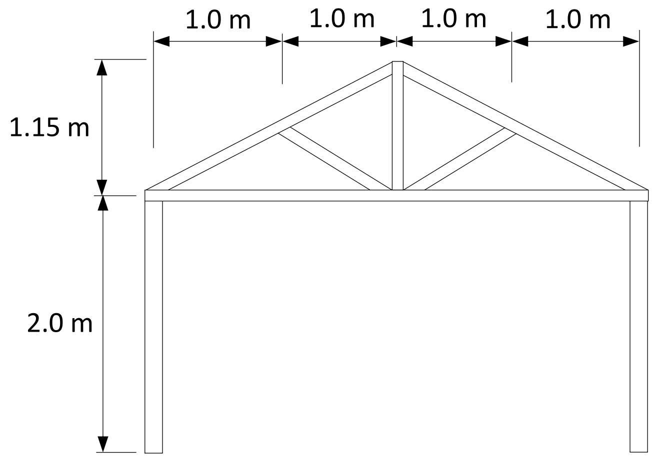
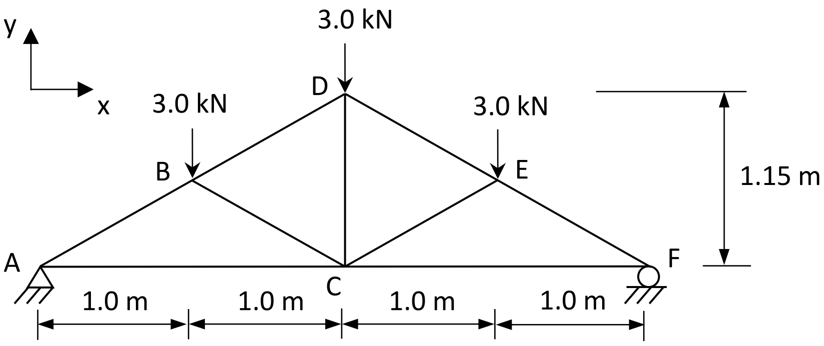
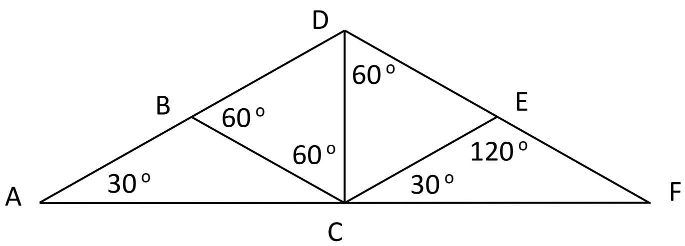
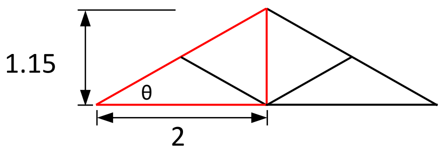


Analysis model
The loading from the roof is transferred to the joints of the truss. All joints are assumed to be pin connected and therefore the members only take axial loads.
The truss is modelled with a pin support on the left and a roller support on the right.
More about analysis models can be found here.



The geometry of the truss was used to find the angles of each member. An example calculation is shown below.



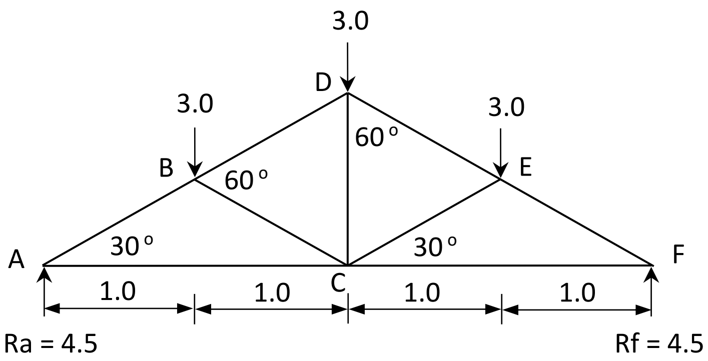


$\tan(\theta) = 1.15/2$
$\theta = \tan^{-1}(1.15/2) = 30^o$
$\sin(30) = 0.5$
$\cos(30) = 0.866$

All forces and dimensions are given in kN and m respectively, unless otherwise stated.

Reactions

Apply the principle of equilibrium to a free body diagram of the entire truss.



Free body diagram

Take moments at the left support:

The sign convention for moments is normally taken as positive clockwise.

Σ M = 0

Rf*4 – 3*1 – 3*2 – 3*3 = 0

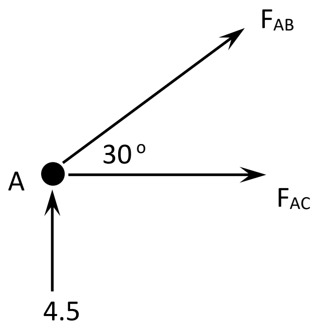
Rf = 18/4 = 4.5 kN

The geometry and loading of the structure is symmetric and therefore the reactions will be equal.

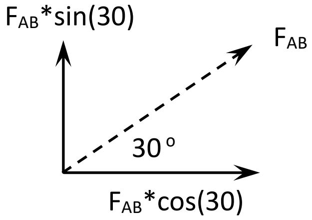
Ra = Rf = 4.5 kN

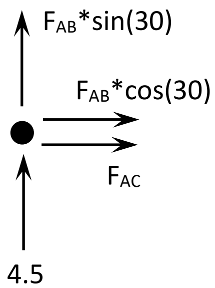
Isolate joint A

Select a joint, in this case Joint A, from the truss which has only two (or less) members with unknown forces and draw a free body diagram showing the forces at the joint.
Write and solve the equations of equilibrium for the joint.
Initially all members with unknown forces are assumed to be in tension therefore the force is directed away from the joint. More information about identifying tension and compression members can be found here.



Resolve forces into vertical and horizontal components





Apply vertical equilibrium: Σ Fy = 0

4.5 + F_{AB}*sin(30) = 0

F_{AB} = - 4.5/sin(30) = - 4.5/0.5

F_{AB}= - 9 kN (compression)

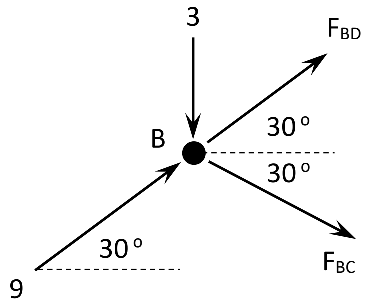
Apply horizontal equilibrium: Σ Fx = 0

F_{AC} + F_{AB}*cos(30) = 0

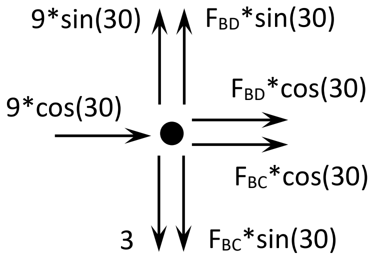
F_{AC} = - (- 9)*cos(30) = 9*0.866

F_{AC} = 7.8 kN (tension)

Isolate joint B



Resolve forces into vertical and horizontal components →



Apply vertical equilibrium: $\Sigma Fy = 0$

$9*\sin(30) + F_{BD}*\sin(30) – 3 – F_{BC}*\sin(30) = 0$

Substitute for sin(30) = 0.5:

$9*0.5 + F_{BD}*0.5 – 3 – F_{BC}*0.5 = 0$

$1.5 + F_{BD}*0.5 – F_{BC}*0.5 = 0$

Rearrange to find an expression for F_{BD}:

$F_{BD}*0.5 = - 1.5 + F_{BC}*0.5$

$F_{BD} = - 3 + F_{BC}$

Apply horizontal equilibrium: $\Sigma Fx = 0$

$9*\cos(30) + F_{BD}*\cos(30) + F_{BC}*\cos(30) = 0$

Divide each term by cos(30):

$9 + F_{BD} + F_{BC} = 0$

Substitute $F_{BD} = - 3 + F_{BC}$:

$9 + (- 3 + F_{BC}) + F_{BC} = 0$

$6 + 2F_{BC} = 0$

$F_{BC} = - 3$ kN (compression)

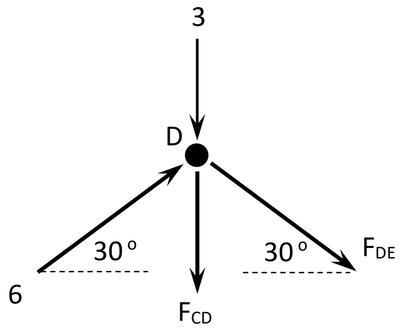
Solve for F_{BD}:

$F_{BD} = - 3 + F_{BC}$

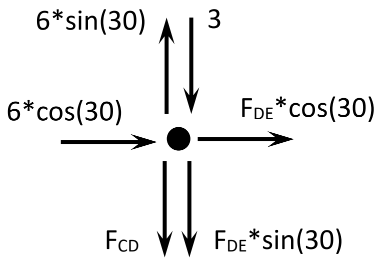
$F_{BD} = - 3 – 3 = - 6$ kN (compression)

An alternative method for solving this joint can be found here.

Isolate joint D



Resolve forces into vertical and horizontal components →



Apply horizontal equilibrium: $\Sigma Fx = 0$

$6*\cos(30) + F_{DE}*\cos(30) = 0$

Divide by cos(30)

$6 + F_{DE} = 0$

$F_{DE} = - 6$ kN (compression)

Apply vertical equilibrium: $\Sigma Fy = 0$

$6*\sin(30) – 3 – F_{CD} – F_{DE}*\sin(30) = 0$

$F_{CD} = 6*\sin(30) – 3 – (- 6)*\sin(30)$

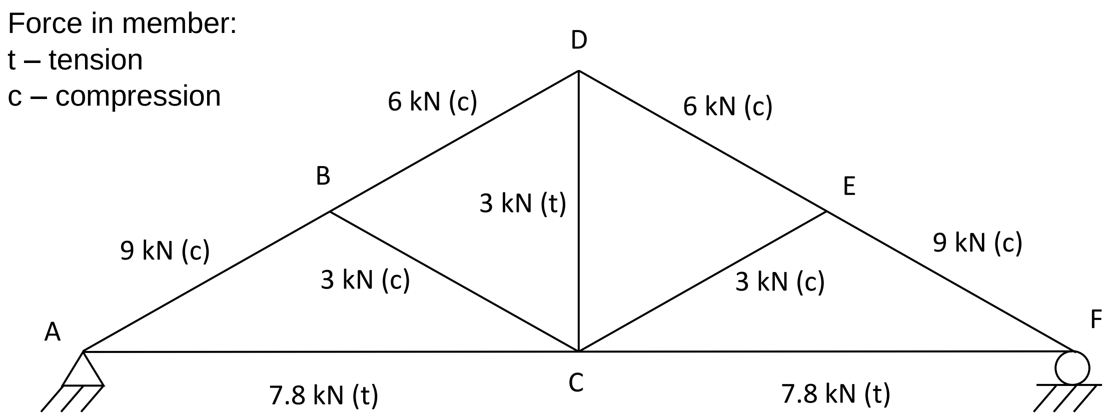
$= 6*0.5 - 3 - (- 6)*0.5$

$F_{CD} = 3$ kN (tension)

Results

Member	Magnitude (kN)	Nature
AB	9	Compressive (strut)
AC	7.8	Tensile (tie)
BC	3	Compressive (strut)
BD	6	Compressive (strut)
CD	3	Tensile (tie)
CE	3	Compressive (strut)
CF	7.8	Tensile (tie)
DE	6	Compressive (strut)
EF	9	Compressive (strut)

The loading and geometry of the structure is symmetric meaning the forces in members to the left of the truss will be the same as the forces in the equivalent member at the right of the truss. This was demonstrated above when the force in member DE was calculated to be the same as the force in member BD. Using this information, the magnitude and nature of the forces in each member can be determined.



Process for nodal analysis:

1. Apply the principle of equilibrium to the entire structure to determine reaction forces.
2. Isolate a joint with only two (or less) members that have unknown forces.
3. Resolve the forces in members at the joint into the x and y directions.
4. Write the equations of equilibrium in the x and y directions for the joint.
5. Solve these equations to identify the unknown forces at the joint.
6. Identify whether the force is tensile or compressive.
7. Continue to isolate each joint in the structure until all unknown forces have been calculated.

Metadata
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