Online information resource for structural mechanics

Moment as a vector cross product

Figure 1 shows a force vector ***F*** in the y direction acting at point A. It has magnitude *F.* (Vector names are in bold)
M is the moment of *F* about the origin of the coordinate axes at O. It is the moment about the z axis. It has magnitude *M.*
r is the position vector for A in relation to O. It has magnitude *r.*

The magnitude of ***M*** is the area of the parallelogram (hatched area) that has sides that are parallel to ***r*** and ***M*** i.e.

$M = r F \sin\theta$

where θ is the angle between the ***M*** and ***r*** vectors.
Note that r $\sin\theta$ is the magnitude of the lever arm for ***F***.

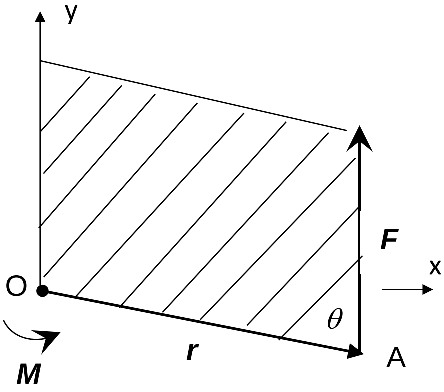

Figure 1 Area vector

The direction of ***M*** is at right angles to the plane defined by ***r*** and ***F*** i.e. it is in the z direction.

In vector notation:

$\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$

The 'x' symbol implies this is a 'cross product' i.e. it represents a vector algebra process that transforms ***r*** and ***F*** into ***M***. The details of this process are not discussed here. The cross product is normally evaluated using software.

Important insight
The important insight from this explanation is that a moment acts in a plane defined by a force vector and a position vector. Its magnitude can be identified as an area in that plane and the direction of the moment vector is at right angles to the plane.

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