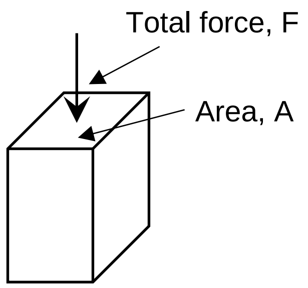
Online information resource for structural mechanics

<u>Stress, strain & deformation</u>

Stress

Stress is the force per unit area.



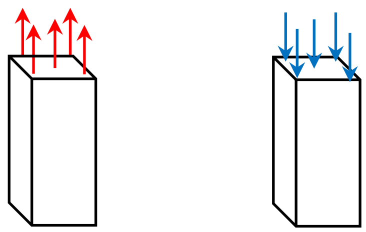
A direct stress, σ, is in a direction that is at right angles to the plane on which it acts.

$\sigma = F/A$

Here we are concerned with stress in axially loaded members and use the term ‘stress’ to mean ‘axial stress’.
Stress is typically given in N/mm^2.

Stress can be:

- Tensile: the force tends to pull away from the surface of the member and increase the length of the member
- Compressive: the force tends to push on the surface and decrease the length of the member



Tensile stress   Compressive stress

Tensile stress is normally considered to be positive and compressive stress negative.

Example 1:

The man in the picture weighs 85kg and the rope has a diameter of 12mm.

a) What stress is exerted on the rope?
b) If the maximum allowable stress that can be carried by the rope is 30N/mm^2 what is the maximum that the man should weigh?

a)
Calculate the applied force, F, on the rope
1kg = 10N, therefore
F = 85*10 = 850N

Calculate the area, A, of the rope
A = π*r^2 = (π*d^2)/4 = (π*12^2)/4 = 113.1mm^2

Calculate the stress
σ =F/A = 850/113.1 = 7.52N/mm^-2

b)
Rearrange the stress equation in terms of the force, F
σ = F/A thus F = σ*A

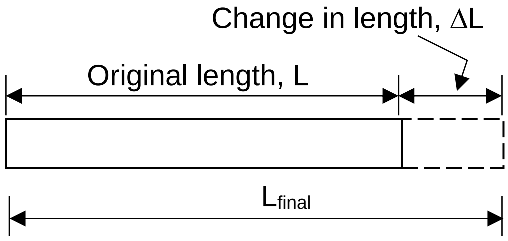
Solve for F using the area calculated in a)
F = σ*A = 30113.1 = 3393N

Convert the force to weight
1N=0.1kg
Maximum weight = 3393*0.1 = 339kg

Strain

Strain is the change in shape of a material in relation to its original shape. It is a dimensionless variable. Here, we consider only axial strain that corresponds to axial stress and refer to it as 'strain'.

Strain, ε, is defined as the ratio of the change in length due to deformation of the material to the original length i.e.
$\varepsilon = \Delta L/L$
where L = initial length and $\Delta L = L_{final} - L$



If the length increases, the strain is tensile. If it decreases the strain is compressive. Corresponding to the sign convention for stress, tensile strain is positive and compressive strain is negative.
The application of stress/load in a material always results in strain. Additionally, strain can be caused by temperature changes.

Example 2:
If the length of the rope supporting the man at the waterfall was 6.0 m before it took his weight, and 6.5 m when he is suspended on it, what is the strain in the rope?

Calculate the change in length
$\Delta L = L_{final} - L = 6.5 - 6 = 0.5\text{m}$

Calculate the strain
$\varepsilon = \Delta L/L = 0.5/6 = 0.08$

Young's modulus of elasticity

A very common relationship between stress and strain is the assumption that the stress is linearly proportional to strain. This means, for example if the stress is doubled the strain also doubles. The relationship for direct stress and strain is:

$$\sigma = E\,\varepsilon$$

where σ is the stress, E is Young's modulus and ε is the strain.
Rearranging this equation in terms of Young's modulus gives:

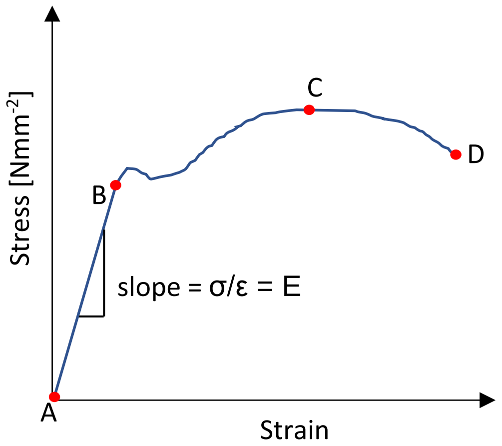
$$E = \sigma/\varepsilon$$

Since strain is dimensionless, it has the same units as stress.
Young's modulus is the stiffness of a material: the higher the modulus, the stiffer the material.

Stress-Strain Relationships

This is a video of a tensile test performed on carbon steel which shows the graph of stress against strain as the load increases to failure. Such a stress-strain graph can provide a lot of information about the properties and behaviour of a material.

Stress-strain curve for mild steel in tension



Stress-strain curve for mild steel

Segment A-B represents linear elastic behaviour where:

- *Elastic* means that when the load is removed, the material returns to its original shape.
- *Linear* means that the slope of the curve is straight. The angle of the line to the horizontal is Young's Modulus $E = \sigma/\varepsilon$

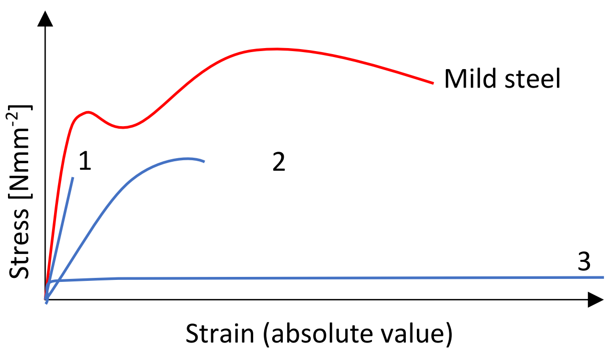
Point B is at the end of the straight line segment A-B. It is the *yield point* beyond which any further loading will cause *plastic deformation*. The main feature of plastic deformation is that when the load is removed, the material will not return to its original shape.

Point C is the *ultimate tensile strength*, the maximum stress that a material can withstand.

The sample will show necking in segment D-E. *Necking* is the rapid localized decrease in cross-sectional area at the part of the section where it will eventually break

Point D is the *fracture point* at which the material breaks.

Stress-strain curves for other materials



Stress strain curve for different materials

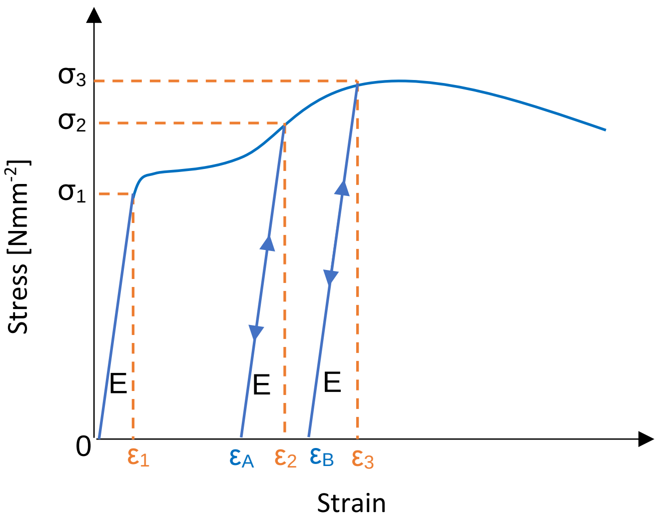
Curve 1: timber – in tension. This can withstand some tensile stresses but is brittle, meaning that it will fail abruptly without large prior deformations. It has no plasticity in tension.

Curve 2: concrete – in compression. This experiences some plastic deformation before failure.

Curve 3: chewing gum. This is an extremely plastic material. It cannot sustain much load but deforms significantly before fracture. It has a very steep linear elastic region, meaning that as soon as any load is applied, the yield point is reached and thus gum does not return to its original shape at all

Strain hardening

Many materials, especially metals and polymers, tend to have both elastic and plastic phases. If the material is loaded beyond the yield point and is then unloaded, some of the deformation is recovered. The slope of the unloading curve is the same as for loading, i.e. the stress/strain ratio = E. This characteristic is used to improve the yield stresses of materials through the process of *strain hardening*. Materials are loaded and unloaded several times beyond their yield point which gradually increases their yield strength.



Strain Hardening Curve for Mild Steel

Strain hardening process:

1. Load stress from 0 → σ_1. This causes strain to increase from 0 → ε_1
2. Unload back to 0. As $\sigma_1 <$ yield stress the strain will return to 0.
3. Load stress from 0 → σ_2. This causes strain to increase from 0 → ε_2
4. Unload back to 0. As $\sigma_2 >$ yield stress the strain will decrease at a rate of E, causing strain to decrease from $\varepsilon_2 \rightarrow \varepsilon_A$
5. Load stress from 0 → σ_3. This causes strain to increase from $\varepsilon_A \rightarrow \varepsilon_3$
6. Unload back to 0. As $\sigma_3 >$ yield stress the strain will decrease at a rate of E, causing strain to decrease from $\varepsilon_3 \rightarrow \varepsilon_B$

At the end of the process, the steel will have a strain of ε_B that cannot be recovered. This is known as *permanent set* or *residual strain*.

This means that an initial section of length $L_{original}$ and yield strength σ_1 can be transformed to a section with a length of $L_{original} + \Delta L$ (where $\Delta L = \varepsilon_B * L_{original}$) with an increased yield strength of σ_3.

Last edited: 14.08.2020
Author: ACR
Keywords: stress, strain, Young's modulus, strain hardening