



Force as a vector, resolution of forces, equilibrium of force at a point,

Scalars and vectors

A *scalar* has the property of magnitude but does not have direction. For example, mass, volume and density are scalars. A *vector* has magnitude and direction. It also has position - at a point or along a line. For example, force, displacement, velocity and acceleration are vectors. The magnitudes of scalars can be added using conventional arithmetic (e.g. $2+2 = 4$) but vectors, because of their direction property are added in a different way.

Vector addition

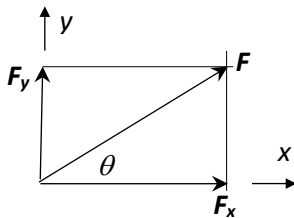


Figure 1 Force vector diagram

In this explanation, the names in bold e.g. \mathbf{F} , represent vectors that have magnitude and direction whereas F (not in bold) represents the magnitude of the vector - a scalar. This distinction is made to help to understand the difference between scalar and vector addition. Using a name in bold to represent a vector is a common notation.

In Figure 1, the lengths of the lines representing the force vectors are scaled to the magnitudes of the vectors (the magnitude of a vector is sometimes denoted as its 'length').

A force vector \mathbf{F}_x acts in the x direction and a force vector \mathbf{F}_y acts in the y direction. \mathbf{F} is a single force vector that is statically equivalent to the combined action of \mathbf{F}_x and \mathbf{F}_y . It is the *resultant* of the two vectors. If the single force \mathbf{F} were applied to a structure, it would have the same effect as applying both \mathbf{F}_x and \mathbf{F}_y .

The magnitude and direction of \mathbf{F} can be calculated from the geometry of the force vector diagram

The magnitude, using Pythagoras' theorem, is: $F = (F_x^2 + F_y^2)^{1/2}$

The direction is: $\theta = \tan^{-1}(F_y/F_x)$

For example, if $F_x = 3.0$ kN and $F_y = 2.0$ kN then:

$$F = (F_x^2 + F_y^2)^{1/2} = \text{sqrt}(2.0^2 + 3.0^2) = 3.6 \text{ kN}$$

$$\theta = \tan^{-1}(F_y/F_x) = \text{atan}(2.0/3.0) = 33.8^\circ$$

This is the process for *vector addition*.

The addition $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$ results in a new vector \mathbf{F} defined, in this example, by a magnitude (3.6 kN) and a direction (33.8°).

Vector addition along a line

If the vectors to be added all lie along the same line, then their magnitudes can be added

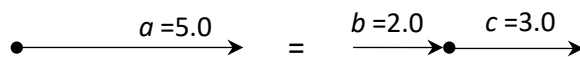


Figure 2 Vector addition along a line

$$a = b + c = 2.0 + 3.0 = 5.0$$

using scalar addition. For example, Figure 2 shows forces with magnitudes $b = 2.0$ and $c = 3.0$ acting along a line. The resultant vector, of magnitude a , is also along that line and the addition of the magnitudes is:

In order to simplify the addition of force vectors it is normal to *resolve* them into the coordinate directions so that their magnitudes can be added in this way.

Vector resolution

Resolution of forces is the inverse of addition. Instead of transforming two vectors into one (addition), one vector is transformed into two (resolution).

The rules for resolving a vector into x and y components are (from the geometry of Figure 1):

$$F_x = F\cos(\theta), \quad F_y = F\sin(\theta)$$

This is one of the most useful relationships in structural mechanics. It is used in [nodal analysis](#) of frames and when calculating the values of moments of forces: see [equilibrium of angled loads](#) and [crane stay application sheet](#).

Example - Equilibrium check at a point

Figure 3(a) shows a set of forces in a plane where one of the forces is not parallel to a coordinate axis. Figure 3(b) shows the 7.5 kN force resolved into the x and y directions. It is clear that the sums of the forces in the x and y directions are zero. Hence equilibrium is satisfied.

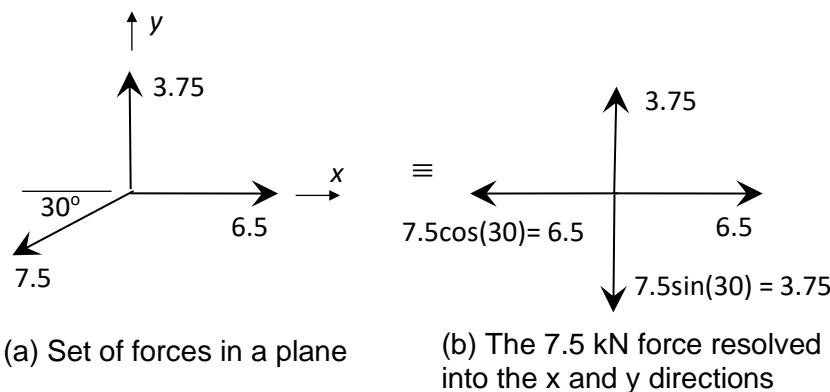


Figure 3 Resolving a force to check for equilibrium

More about vector addition - the parallelogram of forces and the triangle of forces

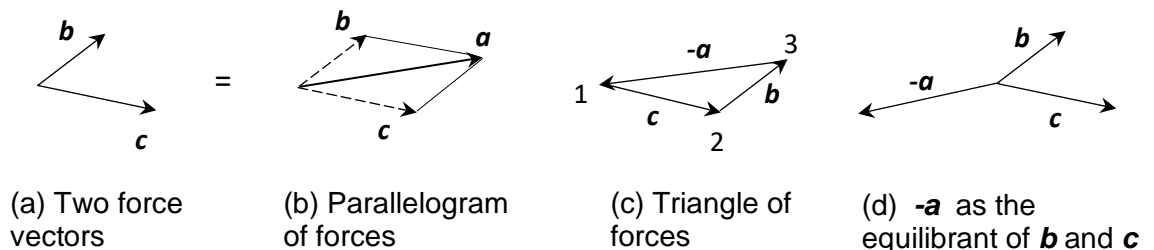


Figure 4 Parallelogram of forces and triangle of forces

Parallelogram of forces

Figure 4(a) shows two force vectors b and c neither of which are parallel to a coordinate axis. Figure 4(b) shows the corresponding parallelogram of forces. The vector a is the resultant of b and c i.e. it is the vector sum of b and c . It is drawn by creating a parallelogram that has sides that are parallel to the directions of b and c . a is the diagonal of the parallelogram. The magnitude and direction of a can be calculated from the geometry of the parallelogram. The parallelogram is introduced here to further explain the concept of force as a vector. It is not normal to work with the geometry of a parallelogram.

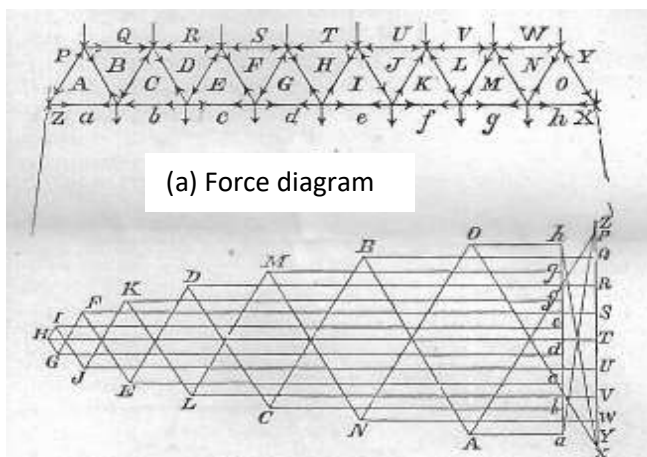
Triangle/polygon of forces

Figure 4(c) is a triangle of forces whose geometry is one half of the parallelogram. It can be drawn follows:

Start at point 1 and draw **c** to scale to establish point 2. From point 2 draw **b** to scale to establish point 3. The line 3-1 is now **-a**, the equal and opposite vector of **a**. **-a** is the *equilibrant* of **b** and **c**, i.e. it is the extra force needed for the point to be in equilibrium. Figure 4(d) shows the forces for equilibrium.

If there are more than three forces at a point, a polygon of forces can be drawn that has the same number of sides as the number of forces at the point. For equilibrium the polygon has to be a closed figure. Drawing a triangle/polygon is a graphical method for vector addition that can be used to implement the principle of equilibrium.

As an alternative to nodal analysis, the forces in the members of a frame can be found graphically by drawing polygons and measuring the lengths of the lines. A composite set of polygons, known as a *reciprocal diagram*, can be drawn for this purpose. Figure 5 shows a reciprocal diagram for a bridge truss from a 19th century textbook on structural mechanics. This type of analysis has been superseded by computer modelling.



(a) Force diagram

(b) Reciprocal diagram



(c) A modern bridge truss

Figure 5 Reciprocal diagram for a bridge truss

Summary

- A vector has magnitude and direction. Force and displacement are vectors.
- Vectors also have position (at a point or along a line) and they satisfy the rules of vector algebra.
- Vector addition results in a vector defined by a magnitude that is not necessarily the scalar sum of the magnitudes of the vectors to be added, and by a direction.
- To make vector addition easier to perform, it is normal to resolve forces that are not in one of the coordinate directions into components in these directions. Scalar addition can then be applied to the magnitudes of the force vectors in the coordinate directions.
- Force vectors are resolved using: $F_x = F\cos(\theta)$, $F_y = F\sin(\theta)$

Metadata

Keywords: Force vectors, vector addition, vector resolution

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Last edited: 11.08.2020