Moment Equilibrium for a Seesaw

Children on a seesaw

Two children play on a seesaw and try to balance it such that the beam is horizontal and both of them are stationary. Assuming that both children weigh the same, they would have to be at an equal distance from the central pivot point to achieve this situation. How is this effect defined in mechanics?

Girls on a seesaw (*"Seeing and Sawing" by Donnie Ray Jones licensed under CC BY 2.0*)

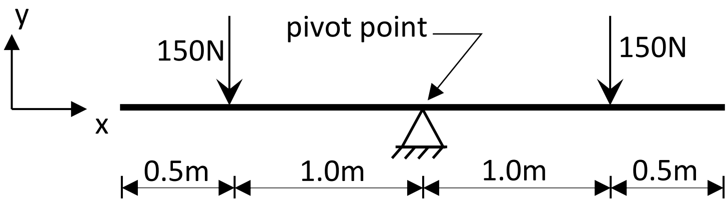

Figure 1: model of the seesaw

Figure 1 is a model of the seesaw that shows the applied forces. If the mass of a child is 15 kg then they each impose a gravity force of 15*10 = 150 Newtons on the beam. The single support at the centre of the beam is the 'pivot point'.
From here on, all forces shown in figures will be in N and dimensions in m, unless stated otherwise.

Definition of a moment

If only one of the children sits on the seesaw, the system will be out of balance and will rotate. The measure of the rotation effect of a force is called a moment. A moment, M, is calculated by multiplying the value of the force, F, by the *lever arm*, d:

$$M = F*d$$

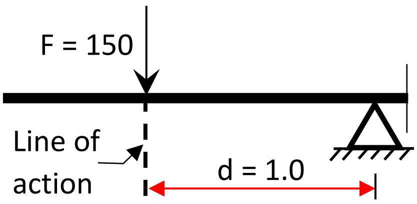

Figure 2: lever arm

The lever arm for the moment of the 150 N force about the pivot point is the perpendicular distance, d, from the pivot to the line of action of the force. In Figure 2 this is equal to 1.0m. Thus, the moment, M = F*d = 150*1 = 150Nm

Moment equilibrium

There are two ways in which moment equilibrium can be defined. For the moments of forces about a point in a structure or part of a structure:

a) the sum of the anticlockwise moments = the sum of the clockwise moments (or vice versa) i.e.:

$$\Sigma M_{anticlockwise} = \Sigma M_{clockwise} \quad \text{or}$$

b) by assigning signs to the moments, their sum will be zero, i.e.:

$$\Sigma M = 0$$

It is normal to define anticlockwise moments as positive and clockwise ones as negative.
The second method is more common in practice and hence will be used in this example.
In Figure 1, moment equilibrium is satisfied because the clockwise moment = -150*1.0 and the anticlockwise moment = 150*1.0. Thus ΣM = -150+150 = 0Nm.

Moment equilibrium with 3 children on the see-saw

What if another child joins the child on the right, at a distance of 0.5m from the pivot point? To maintain equilibrium, the child on the left has to move further away from the pivot point. But how far?

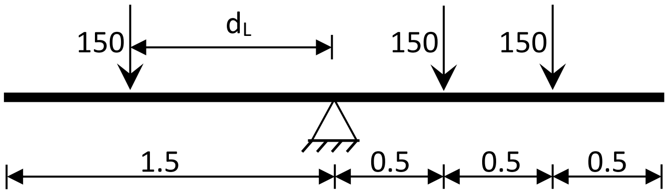

Figure 3: model of the seesaw with an added force

Take moments about the pivot and apply moment equilibrium:

$\Sigma M = 0$

$150 * d_L - 150 * 0.5 - 150 * 1 = 0$

$150 * d_L = 225$

Thus, $d_L = 1.5$m

If the third child sat at the pivot point instead, their weight would have no effect on the motion of the see-saw because it would have no lever arm about the pivot.

Equilibrant and resultant of the forces

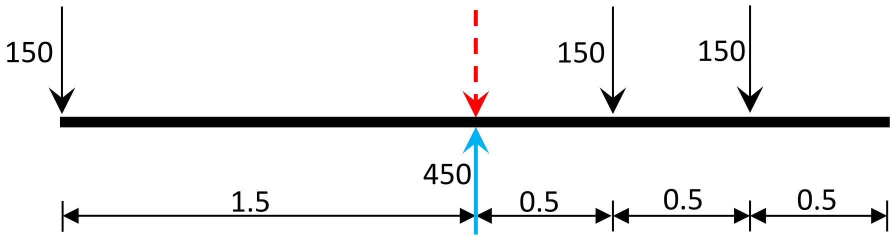

Figure 4: free body diagram of the seesaw with 3 children

Figure 4 shows a free body diagram for the seesaw. The resultant of the 3 forces is equal to: 3*150 = 450N pointing downwards, positioned at the pivot point. This is indicated by the red dotted arrow in Figure 4.

For vertical equilibrium to be satisfied, the reaction force at the pivot also has to be: 3*150 = 450N but pointing upwards. This is indicated with the blue arrow in Figure 4.

The reaction force is the equilibrant of the resultant of the three forces. This means that it is equal and opposite to the resultant, i.e. that it has the same magnitude (450N) but points in the opposite direction (upwards).

Concept summary

- A moment is the rotational effect of a force
- Moment = Force x lever arm
- The lever arm is the perpendicular distance from the line of action of a force to a point
- Anticlockwise moments are typically taken as positive, clockwise ones as negative
- Moment equilibrium: $\sum M = 0$
- Moment equilibrium can be applied about any point on a structure

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