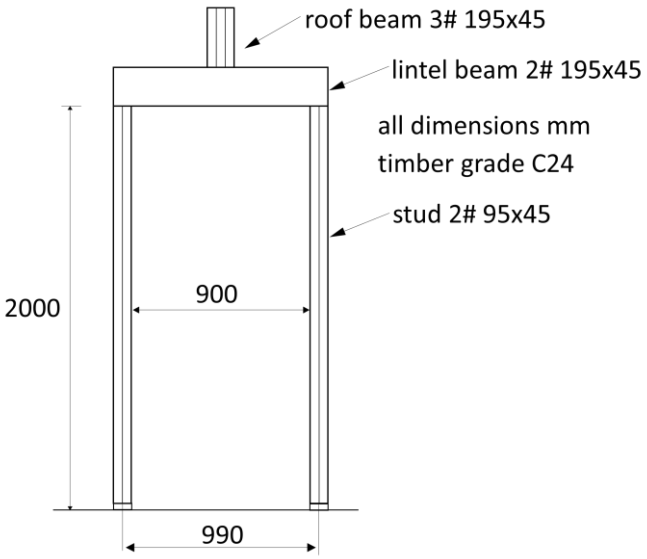
Vertical, Horizontal and Moment Equilibrium for a Beam



Engineering model of door frame

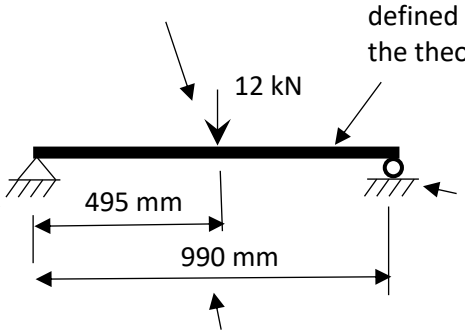
The image shows a timber door frame for a renovation in a house. It is part of a timber 'stud frame' that provides the structure for a partition.
The lintel here supports a heavily loaded roof beam and is therefore has a larger than normal size in order to have sufficient strength to span between the supports.
This example shows how the principle of equilibrium is applied to calculate the reactions on the beam.

Analysis model of the lintel beam

The analysis model is a mathematical representation of the behaviour of the beam

Loading There is a 12 kN vertical load at the centre of the beam

The *behaviour* of the structure is defined along a line -according to the theory of bending



This is a *roller support*, a type of *restraint*, that does not allow vertical movement.

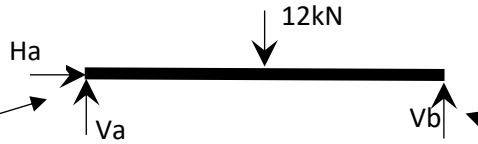
This is a *pin support*, a type of *restraint, that* does not allow vertical or horizontal movement.

Geometry The span of the beam is 9990 mm

Free body diagram for the lintel beam

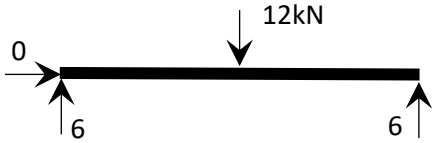
To create a *free-body diagram* of a structure, the restraints are replaced by forces.

At the pin restraint there can be horizontal and vertical reactions



At the roller restraint there can be only a vertical reaction force

Apply the principle of equilibrium for horizontal and for vertical forces



Horizontal forces
The sum of the horizontal forces to the right is equal to the sum of the horizontal forces to the left

$Ha = 0$

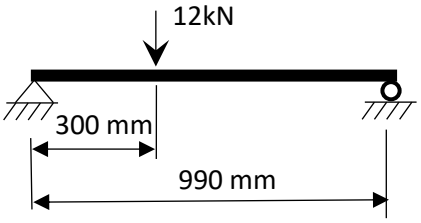
Since there is no horizontal load, the single horizontal reaction must be zero.

Vertical forces
The sum of the upward forces is equal to the sum of the downward forces

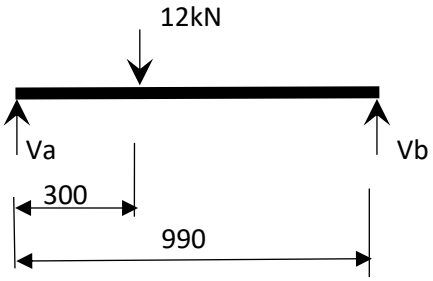
$Va + Vb = 12$

Due to symmetry $Va = Vb = 6$

Calculating reactions when the load in not at the centre
Use of the principle of moment equilibrium



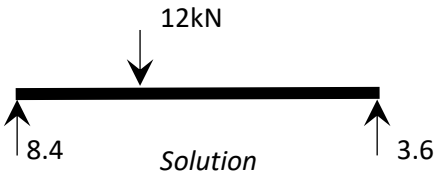
This model shows the roof load at a position 300 from the left-hand support. In order to find the vertical reactions the principle of moment equilibrium must be used.



Free body diagram
For moment equilibrium the sum of the clockwise moments about any point is equal to the sum of the anticlockwise moments

Take moments about the left support
Σclockwise = Σanticlockwise
12*300 = Vb*990 Vb = 12*300/990 = 3.6 kN
(The lever arm for the 12 kN load is 300 mm and that for the right-hand reaction is 990 mm.)

Apply vertical equilibrium
Σup = Σdown Va + 3.6 = 12, Va = 8.4kN



Solution

Process for calculating reactions:

1. Draw a free body diagram for the structure i.e. replace the restraints by forces.
2. Write the equilibrium equations for horizontal forces, for vertical forces and for moment about a point. For moment equilibrium, use a point through which one of the reactions acts.
3. Solve for the unknowns.

Definitions

moment of a force is the value of the force times the lever arm
lever arm is the perpendicular distance from the point about which the moment is being taken to the line of action of the force.
free body diagram a form of the analysis model that shows support reactions rather than support conditions.
restraint the definition of a support condition for an analysis model.

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