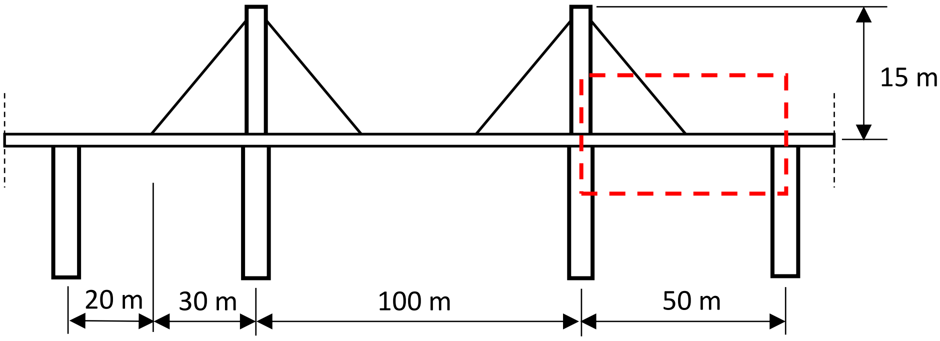
Online information resource for structural mechanics

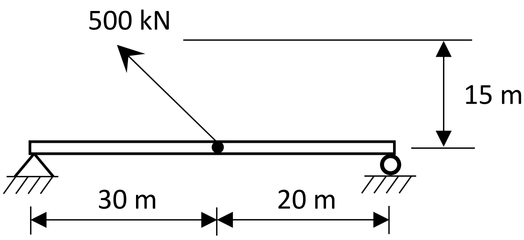
Equilibrium with loads at an angle on a beam

Structures are not only subjected to vertical loading and often have forces applied at different angles. This means the supports of the structure must be able to resist both the vertical and horizontal components of the applied load to satisfy the equilibrium conditions. The cables on a cable stay bridge exert an angled force on the deck due to the tensile force they carry.

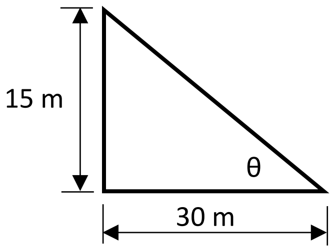
Figure 1 Erskine Bridge
(Matthew Boyle, licensed under (CC BY-NC 2.0)



Analysis of angled load on right span



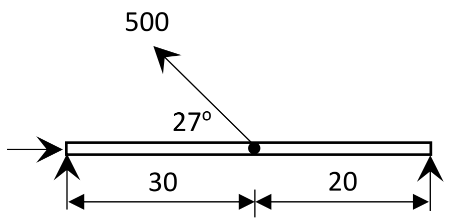
Analysis model
Consider only the shorter span to the right of the bridge. For simplicity, it is modelled as a beam with a pin support on the left and a roller support on the right in order to analyse the effect of the force produced by the tension cable.



The geometry of the bridge was used to find the angle of the cable.

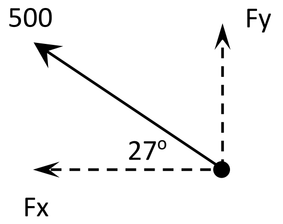
$\tan(\theta) = 15/30$
$\theta = \tan^{-1}(15/30) = 27^{o}$

All forces and dimensions are given in kN and m respectively, unless otherwise stated.



Forces on the structure

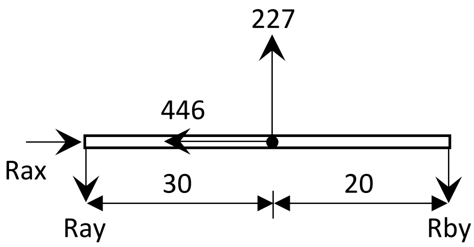
Free body diagram (FBD) of the beam.
The pin support provides resistance to the horizontal component of the force from the cable.



Resolution of force

For the equilibrium equations to be applied, the force must be resolved into its horizontal and vertical components.

Fy = 500*sin(27) = 227 kN
Fx = 500*cos(27) = 446 kN



Reactions

The point load must be balanced by the two end supports.

Apply moment equilibrium

Sum the moments of the forces about the left hand support
As the line of action of the horizontal component passes through the left support, it creates no moment about that point.
Σ M = 0
227*30 + Rby*50 = 0
Rby = - (227*30)/50 = - 136 kN
i.e. the negative sign indicates that the reactions are downward as the cable is pulling up on the deck.

Apply vertical equilibrium: Σ Fy = 0
Ray + Rby + 227 = 0
Ray -136 + 227 = 0
Ray = - 91 kN

Apply horizontal equilibrium: Σ Fx = 0
Rax – 446 = 0
Rax = 446 kN

Note: The horizontal component of force will result in an axial thrust in the bridge deck which would be an important issue to consider during the bridge's design.

Process:

1. Determine the angle of the line of action of the applied force
2. Draw a free-body diagram for the structure.
3. Resolve the force into its x and y components.
4. Write the equations of equilibrium for the structure.
5. Solve these equations to determine the reaction forces.

Metadata

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