



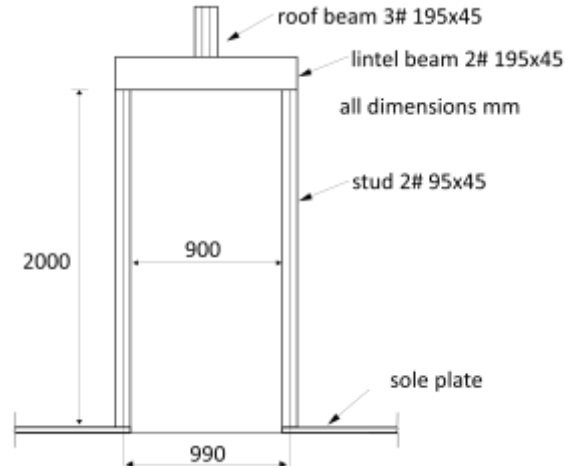
## Bending moments for a lintel beam

*Basic principle - moment equilibrium.* The sum of the moments of forces about a point is zero for: (a) a structure, (b) a part of a structure and (c) at an interface between parts of a structure.

### Vertical forces on a beam



Door lintel beam supporting a roof beam



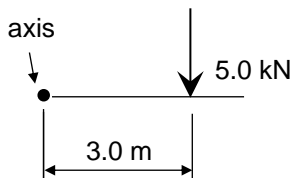
Engineering model of a door frame

### Definitions

**moment** the tendency of a force to rotate an object; the value of a moment about an axis/point is the product of the value of the force times the lever arm.

**lever arm** (for a moment of a force) the perpendicular distance from the axis to the line of action of the force.

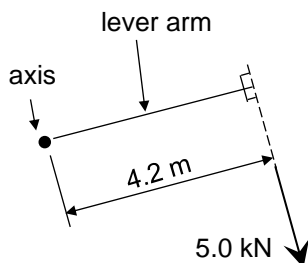
The sign convention for moments is normally: positive anti-clockwise



### Calculating the value of a moment

The axis is at right angles to the plane of the page. The lever arm is 3.0m and therefore the moment of the 5kN force about the axis is:

$$M = \text{force} \times \text{lever arm} = 5.0 \times 3.0 = 15.0 = +15.0 \text{ kNm}$$



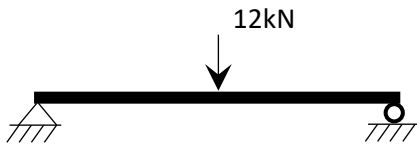
### Force not vertical

Here the line of action of the force is not vertical. It is extended (dotted line) so that the lever arm can be shown at right angles to the line of action. The moment is:

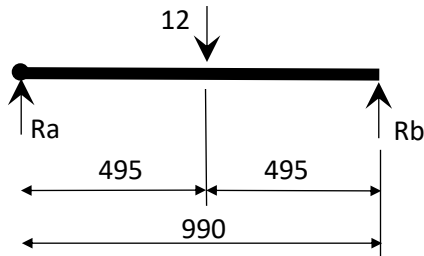
$$M = 5.0 \times 4.2 = +22.0 \text{ kNm}$$

Normally forces are resolved into the vertical and horizontal directions before moments are calculated.

## Use moment equilibrium to calculate the reactions on a beam



Analysis model of the lintel beam example



### Calculate the values of the reactions

Take moments about the left-hand end:

$\Sigma$  clockwise moments =  $\Sigma$  anticlockwise moments  
( $\Sigma$  means 'sum of')

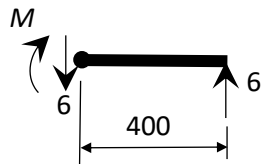
$$12 \cdot 495 = R_b \cdot 990, R_b = 12 \cdot 495 / 990 = 6 \text{ kN}$$

Vertical equilibrium  $R_a + R_b = 12$

$$R_a = 12 - R_b = 12 - 6 = 6 \text{ kN}$$

## Define bending moment

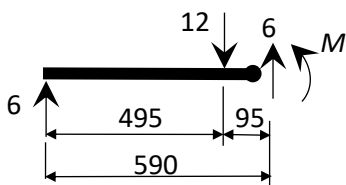
Consider a section 400 mm from the right end of the beam



### Free body diagram (FBD) of part to the right of the section

Take moments at the section:

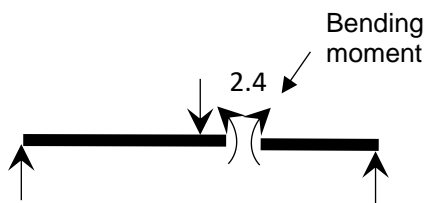
$$M = 6 \cdot 400 = 2400 \text{ kNmm} = 2.4 \text{ kNm}$$



### FBD of part to the left of the section

Take moments at the section:

$$6 \cdot 590 - 12 \cdot 95 = M = 2400 \text{ kNmm} = 2.4 \text{ kNm}$$



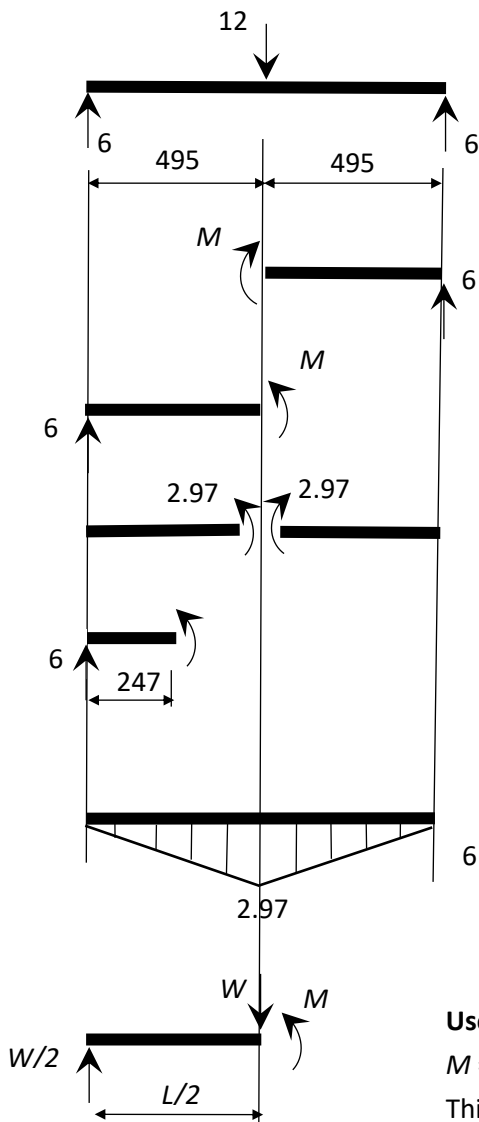
### Forces at a section

(The shear forces at the section are not shown.)

This is the *bending moment* - an internal force action in the member. It can be thought of as a pair of equal and opposite moments at a section.

The value of a bending moment at a section of a beam is the sum of the moments of forces on the part of the beam to one side of the section.

## Draw the bending moment diagram



### FBD of the beam

#### Section at centre of beam

FBD of part of the beam the right of the section.  $M = 6 \times 0.495 = 2.97$  kNm

FBD of part of the beam to the left of the section.  $M = 2.97$  kNm

Bending moment at the centre of the beam

Bending moment at the quarter point  $M = 6 \times 247 = 1.46$  kNm i.e. 0.3 that at the centre, i.e. bending moment is linear.

#### Bending moment diagram

*Sign convention:* The bending moment is drawn on the tension side of the beam. This is the normal convention in structural engineering.

More about sign conventions

#### Use variable names

$$M = (W/2) \times (L/2) = WL/4$$

This is a well-known expression for the maximum bending moment in a centrally loaded simply supported beam.

## Definition

**Bending moment** an internal moment (force action) at a section of a beam that acts about an axis that is in the plane of the section. The value of a bending moment is the sum of the moments of the forces on the beam on one side of the section.

## Failure in bending

The reason for needing to know the value of bending moments is that this is the internal action in a beam that can cause it to fail in bending.

## Metadata

Keywords: Moment equilibrium, free body diagram, bending moment, bending moment diagram.

Author: I MacLeod

Last edited: 24.08.2020